

① Kittel 6.1

$$f(\epsilon) \stackrel{(2)}{=} \frac{1}{\lambda^{-1} e^{\epsilon/\tau} + 1}, \quad \lambda = e^{\mu/\tau}$$

$$\frac{df}{d\epsilon} = \frac{-1}{(\lambda^{-1} e^{\epsilon/\tau} + 1)^2} \cdot \lambda^{-1} e^{\epsilon/\tau} \cdot \frac{1}{\tau}$$

If $\epsilon = \mu$ then $\lambda^{-1} e^{\epsilon/\tau} = e^{-\mu/\tau} e^{\mu/\tau} = 1$, so

$$-\left. \frac{\partial f}{\partial \epsilon} \right|_{\epsilon=\mu} = \frac{1}{(1+1)^2} \cdot \frac{1}{\tau} = \frac{1}{4\tau}, \quad \text{QED}$$

② Kittel 6.2

if $\epsilon = \mu + x$ then $\lambda^{-1} e^{\epsilon/\tau} = e^{-\mu/\tau} e^{(\mu+x)/\tau} = e^{x/\tau} \approx 1 + \frac{x}{\tau}$ for $x \ll \tau$

$$\text{Then } f(\mu+x) = \frac{1}{(1+\frac{x}{\tau})+1} = \frac{1/2}{1+\frac{x}{2\tau}} = \frac{1}{2} \left(1 + \frac{x}{2\tau}\right)^{-1} \approx \frac{1}{2} \left(1 - \frac{x}{2\tau}\right)$$

Hence, setting $x = \delta$ and then $x = -\delta$:

$$f(\mu+\delta) = \frac{1}{2} - \frac{\delta}{4\tau}, \quad f(\mu-\delta) = \frac{1}{2} + \frac{\delta}{4\tau}$$

$$\text{So } 1 - f(\mu-\delta) = 1 - \left(\frac{1}{2} + \frac{\delta}{4\tau}\right) = \frac{1}{2} - \frac{\delta}{4\tau} = f(\mu+\delta), \quad \text{QED}$$

③ Kittel 6.3

(a) The allowed states of the orbital are

